Programming Languages as Notations

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About me: oneliner-izer talk

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- **Claim**: it's possible to write any Python program as one line of code

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Today: notations $\leftrightarrow$ programming languages

Visual notations

Visual notations

Lots of notations used in CS, math, and science are highly visual.

Visual notations in computer science

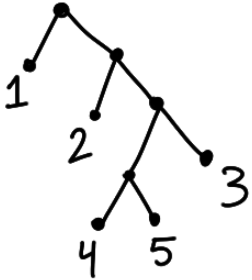
Binary tree.

Visual notations in computer science

Binary tree.

```
Tree(1,
     Tree(2,
          Tree(Tree(4, 5),
               3)))
```

→



Visual notations in computer science

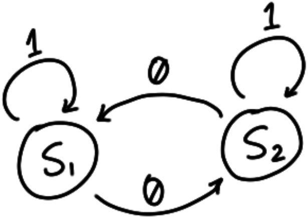
State machine.

Visual notations in computer science

State machine.

```
[
 Transition(S1, 0, S2),
 Transition(S1, 1, S1),
 ...
]
```

→



Visual notations in computer science

Boolean circuits.

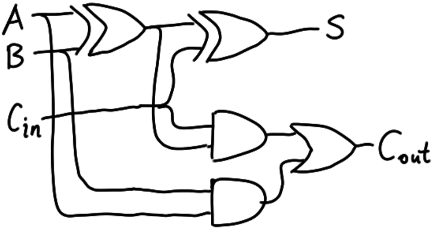
Visual notations in computer science

Boolean circuits.

C_{out} = `Or(And(`A`, `B`),`
`And(`C_{in}`, Xor(`A`, `B`)))`

S = `Xor(Xor(`A`, `B`), `C_{in}`)`

$\rightarrow$



Visual notations beyond computer science

Protein signalling pathways.

Visual notations beyond computer science

Protein signalling pathways.

```
A inhibits B
A inhibits C
B activates C
C activates D
D activates B
```

Visual notations beyond computer science

Protein signalling pathways.

```
A inhibits B
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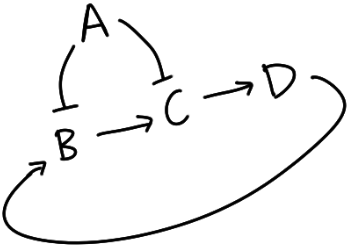

Visual notations beyond computer science

Feynman diagrams.

Visual notations beyond computer science

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- Quantum electrodynamics

Visual notations beyond computer science

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- Sample problem: electron scattering

Visual notations beyond computer science

Feynman diagrams.

- Quantum electrodynamics
- Sample problem: electron scattering

"The formalism was notoriously cumbersome, an algebraic nightmare of distinct terms to track and evaluate. . . Individual contributions to the overall calculation stretched over four or five lines of algebra."

– David Kaiser, in "Physics and Feynman's Diagrams," *American Scientist*, volume 93.

Visual notations beyond computer science

Feynman diagrams.

$$e^2 \int \int K(3,5)K(4,6)\gamma_\mu \delta(s_{56}^2)$$
$$\gamma_\mu K(5,1)K(6,2)d^4x_5 d^4x_6$$

Visual notations beyond computer science

Feynman diagrams.

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Visual notations beyond computer science

Feynman diagrams.

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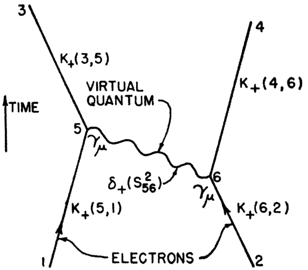

Richard Feynman, *Space-Time Approach to Quantum Electrodynamics*, 1949.
David Kaiser, "Physics and Feynman's Diagrams", *American Scientist* volume 93, 2005.

Visual notations for computer science?

Visual notations for computer science?

```
dbm = 0
for dim in _DAYS_IN_MONTH[1:]:
    _DAYS_BEFORE_MONTH.append(dbm)
    dbm += dim
del dbm, dim

def _is_leap(year):
    "year -> 1 if leap year, else 0."
    return year % 4 == 0 and (year % 100 != 0 or
                              year % 400 == 0)

def _days_before_year(year):
    "year -> number of days before January 1st of year."
    y = year - 1
    return y*365 + y//4 - y//100 + y//400

def _days_in_month(year, month):
    "year, month -> number of days in that month in that year."
    assert 1 <= month <= 12, month
    if month == 2 and _is_leap(year):
        return 29
    return _DAYS_IN_MONTH[month]
```

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→ **What visual equivalent?**

Visual notations for computer science?

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`snakefood` visualizes dependencies in Python codebases:

Visual notations for computer science?

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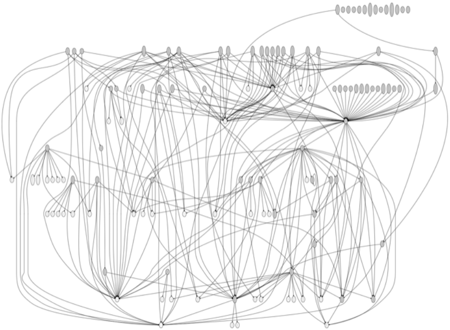

Flask - http://grokcode.com/864/snakefooding-python-code-for-complexity-visualization/

Visual notations for computer science?

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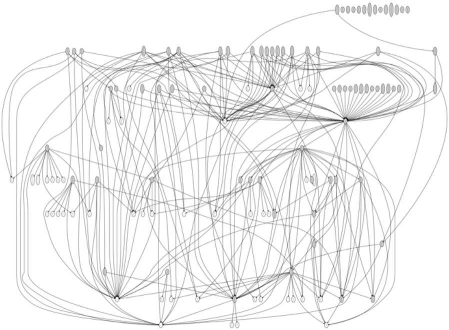

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...but can we have a notation for the *entire language*?

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Lambda calculus is:

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Lambda calculus is:

- A formalization of computation, devised by Alonzo Church around 1935

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Basic design: inputs flow to outputs, passing through functions

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The identity function:

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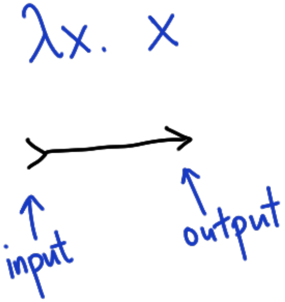

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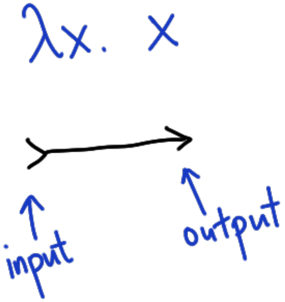

Multiple arguments, and application:

Example: circuitry for lambda calculus

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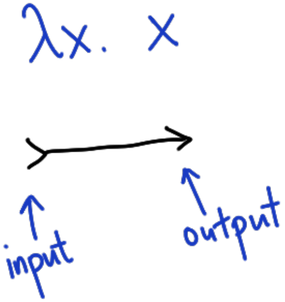

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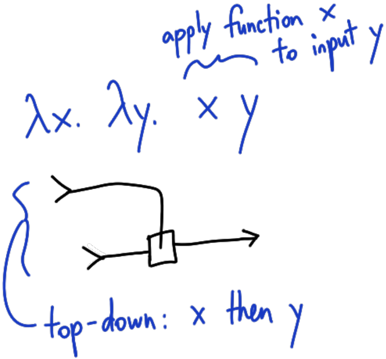

Example: circuitry for lambda calculus

Example execution:

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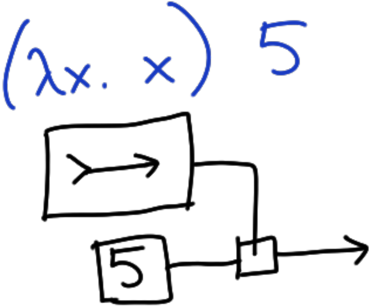

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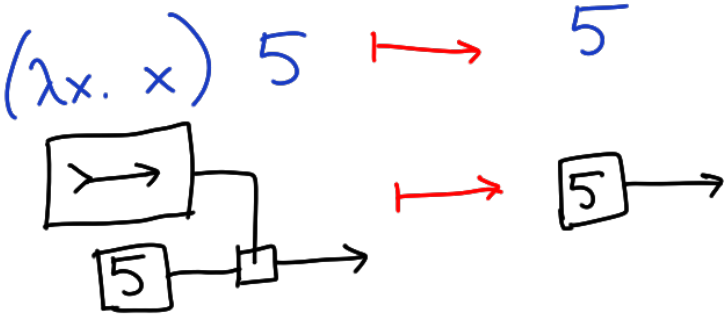

Example: circuitry for lambda calculus

Building Boolean logic:

Example: circuitry for lambda calculus

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"true" $\lambda x.\ \lambda y.\ x$

"false" $\lambda x.\ \lambda y.\ y$

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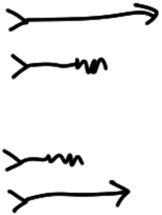
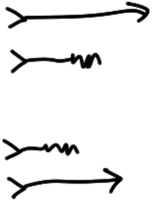


Example: circuitry for lambda calculus

Building Boolean logic:

"true" $\lambda x.\ \lambda y.\ x$

"false" $\lambda x.\ \lambda y.\ y$

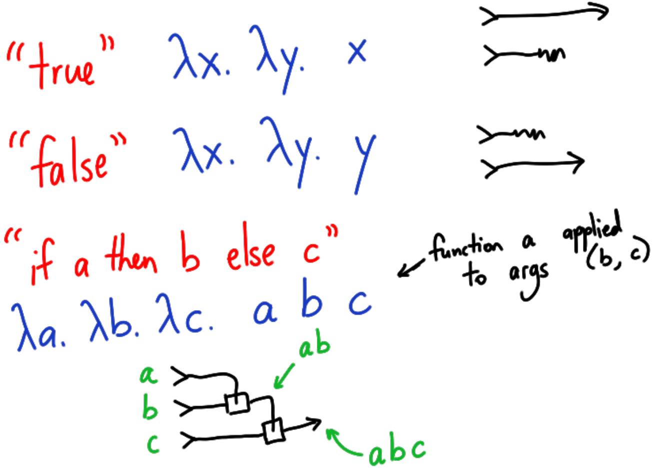


"if a then b else c"

$\lambda a.\ \lambda b.\ \lambda c.\ a\ b\ c$ ← function a applied to args (b, c)

Example: circuitry for lambda calculus

Building Boolean logic:



Example: circuitry for lambda calculus

"if true then 1 else 0"

$(\lambda a. \lambda b. \lambda c.\ a\,b\,c)\ (\lambda x. \lambda y.\ x)\ 1\ 0$

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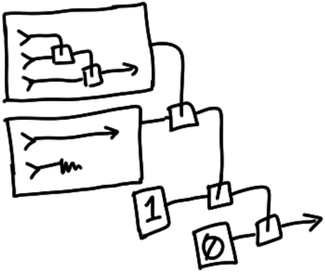

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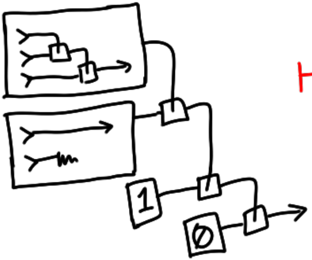

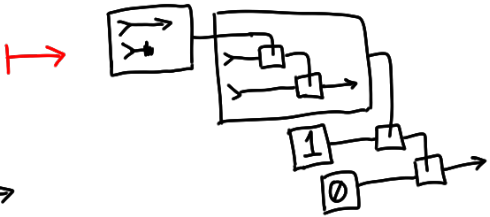

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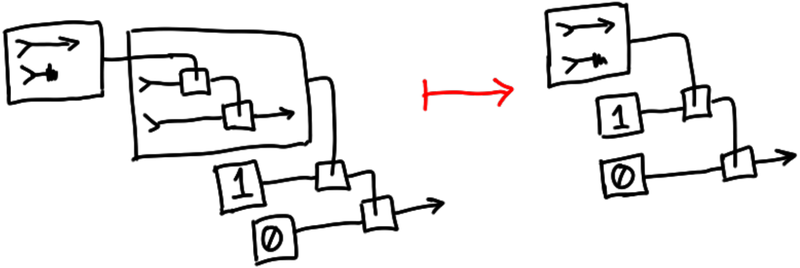

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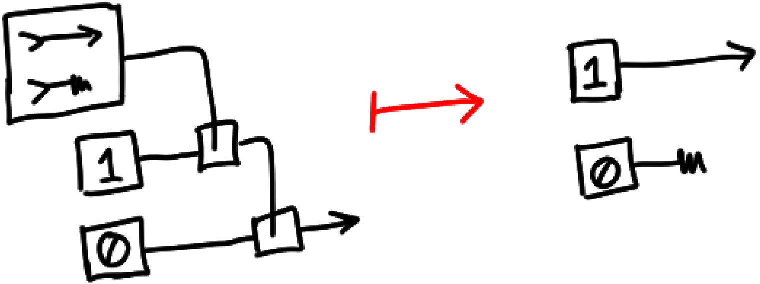

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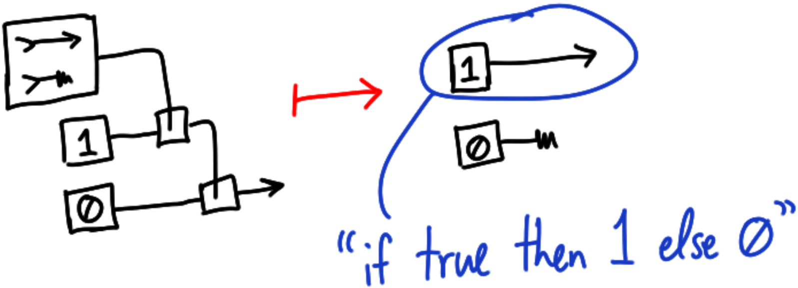


Example: circuitry for lambda calculus

Linked lists

first =

second =

pair =

Example: circuitry for lambda calculus

Linked lists

first = λp. p true

second = λp. p false

pair = λa. λb. λx. if x then a else b

Example: circuitry for lambda calculus

Linked lists

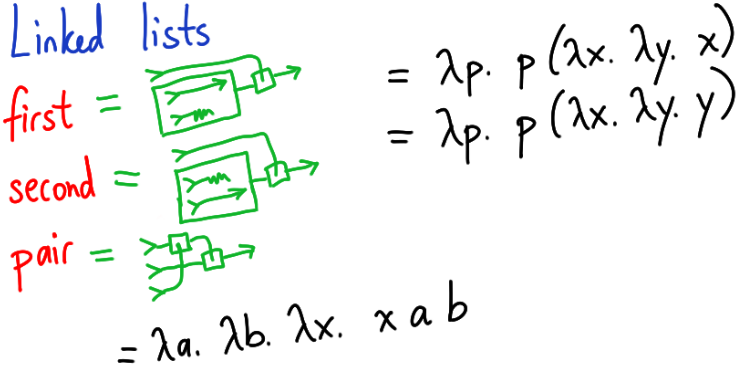
$$\text{first} = \lambda p.\ p\ \text{true} = \lambda p.\ p\ (\lambda x.\ \lambda y.\ x)$$

$$\text{second} = \lambda p.\ p\ \text{false} = \lambda p.\ p\ (\lambda x.\ \lambda y.\ y)$$

$$\text{pair} = \lambda a.\ \lambda b.\ \lambda x.\ \text{if } x \text{ then } a \text{ else } b$$

$$= \lambda a.\ \lambda b.\ \lambda x.\ x\ a\ b$$

Example: circuitry for lambda calculus



Example: circuitry for lambda calculus

Omega combinator – applies input to itself:

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$$\omega = \lambda x.\ (x\ x)$$

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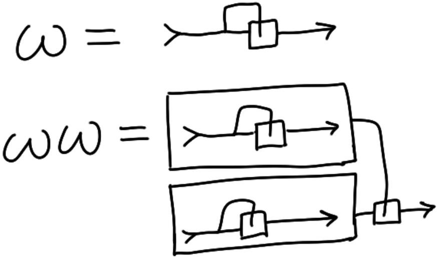

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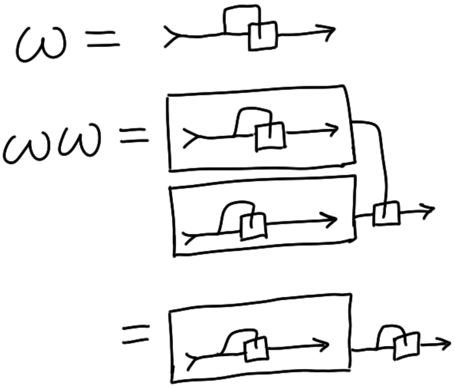

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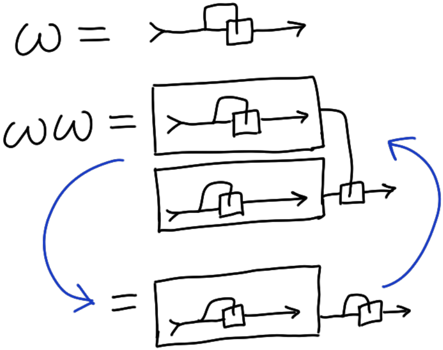

Example: circuitry for lambda calculus

Fixed point combinator:

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$$Y\ f = f\ Y\ f$$

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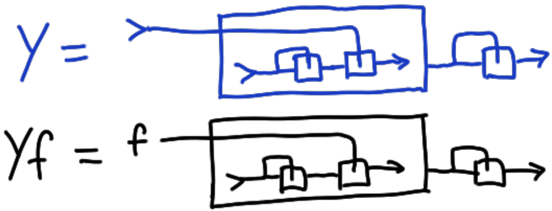

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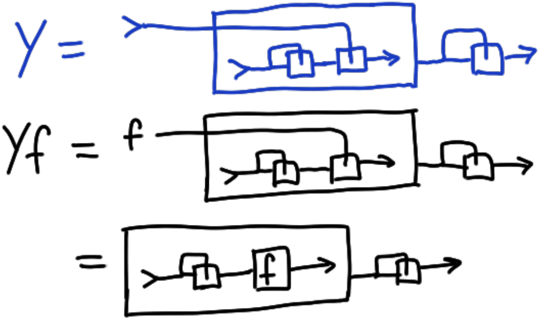

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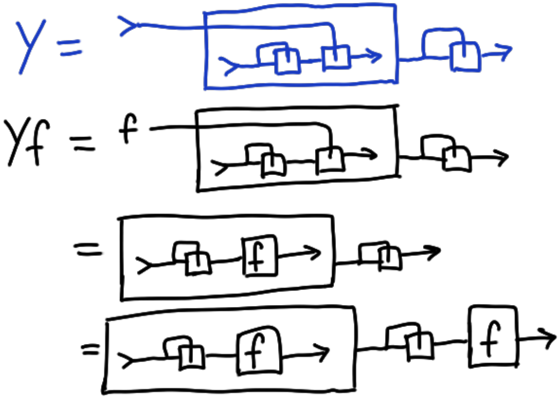

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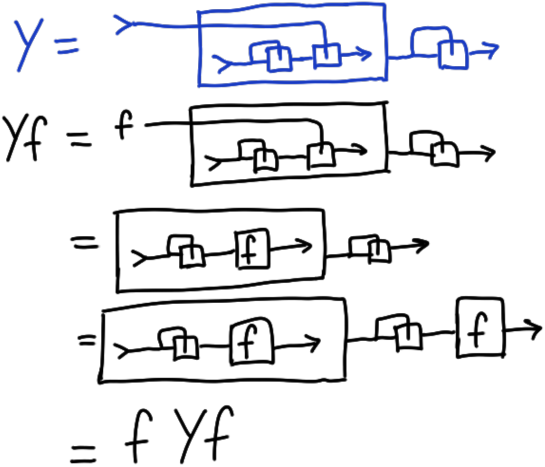

Suggested further exercises

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- map, reduce, filter for the list implementation

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- map, reduce, filter for the list implementation
- combinator puzzles in *To Mock a Mockingbird*

Similar previous work

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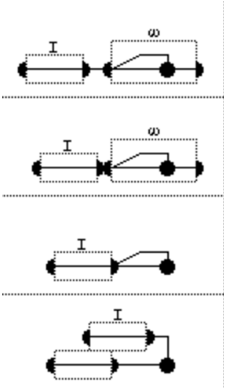
To Dissect a Mockingbird



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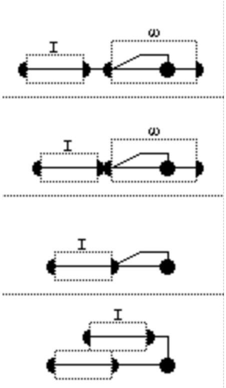
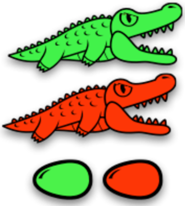


Alligator Eggs game



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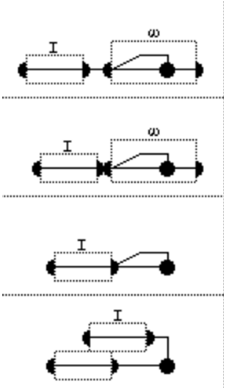
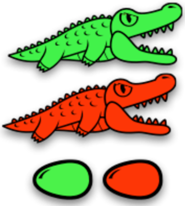
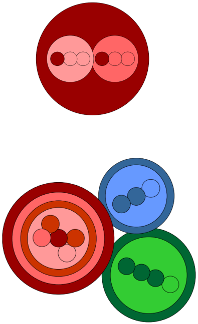


Alligator Eggs game



Visual Lambda Calculus, bubble notation and GUI



Notations as abstractions

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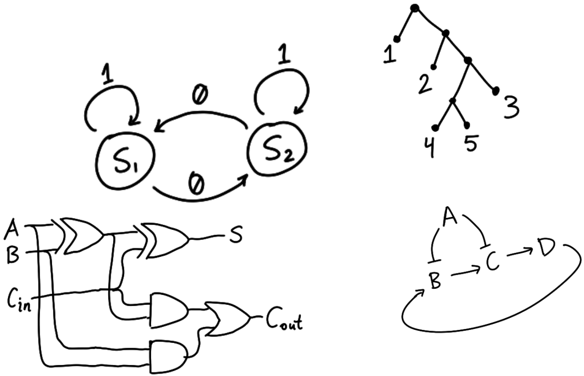

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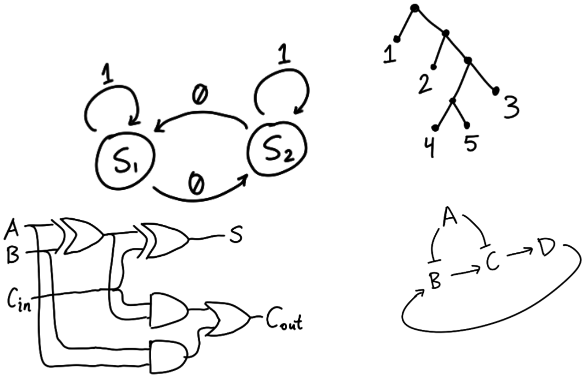

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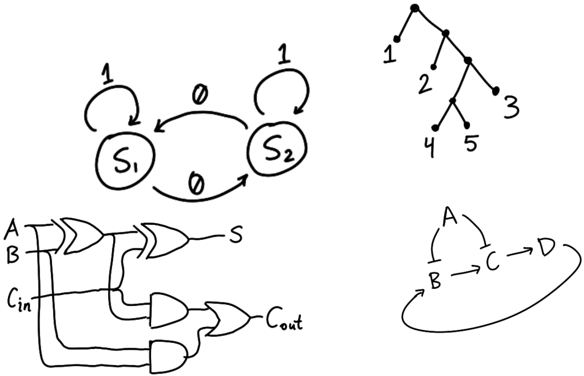

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Abstractions trade **freedom** for **specificity**.

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    - both **sequential composition** and **parallel composition** of code

One final similarity between notations and programming languages...

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sometimes you get into wars about which ones are right!

Notation wars

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"Students who had found quaternions quite hopeless could understand my vectors very well."

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Maybe having many standards is the necessary price of **innovation**.

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Thanks!

`@csvoss`